Problem 1. Let $A$ be a hyperbolic linear map. Then if $\|A - B\|$ is sufficiently close to zero, $B$ is also hyperbolic. ($\|A\| = \sup_{\|v\|=1} \|Av\|$.)

Problem 2. In Prop 3.2 in class we stated that for any norm $\| - \|$ on $\mathbb{R}^n$ and any $\epsilon > 0$, there exists a $C_\epsilon > 0$ such that for all $v \in \mathbb{R}^n$, $\|A^m v\| \leq C_\epsilon (r(A) + \epsilon)^m \|v\|$. Prove that this statement does not hold without the $\epsilon$ by examining $\begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}$.

Problem 3. Let $f : S^1 \to \mathbb{R}$ be a continuous expanding map; that is, suppose that there is some $\lambda > 1$ and some $\delta > 0$ such that for any $x \neq y$ in $S^1$ with $d(x, y) < \delta$,

$$d(fx, fy) > \lambda d(x, y).$$

Let $f_*$ be the homomorphism induced by $f$ on $H_1(S^1) \cong \mathbb{Z}$. $f_* : \mathbb{Z} \to \mathbb{Z}$ is clearly determined by a single number, $f_*(1)$; this is the degree of $f$.

Prove that the degree of a continuous expanding map on the circle is $> 1$ in absolute value.

Problem 4. Let $f : S^1 \to \mathbb{R}$ be a continuous expanding map with degree $m \geq 2$. Prove that $f$ has a fixed point on $S^1$.

Problem 5. Let $f : S^1 \to \mathbb{R}$ be a continuous expanding map with degree $m \geq 2$. Prove that for any $n$ there is a decomposition of $S^1 = [0, 1]/(0 \sim 1)$ into $m^n$ disjoint half-open intervals (left endpoint closed, right endpoint open) denoted $I_i^n$ ($1 \leq i \leq m^n$) so that

$$f(I_i^n) = I_j^{n-1}$$

for some $j$ depending on $i$.

(Hint: do this for $E_m$ first, and then try to adapt the result to the more general setting).
Note: Problems 3, 4, 5 are the first steps in an alternate proof of a structural stability result we will prove for expanding maps of the circle, which will appear as Thms 9.1 and 9.4 in class. This set of problems will be continued on the next problem set.

**Problem 6.** Let $T_A$ be a hyperbolic total automorphism. In class (Prop 5.8) we proved/will prove that $T_A$ is topologically mixing, i.e. that for any open $U, V$ there exists some $N(U, V)$ such that for all $n > N$, $T^n_A(U) \cap V \neq \emptyset$.

Suppose now that $U, V$ are balls of radius $\epsilon$ around points in $\mathbb{T}^2$. Find some $N^*$ which works as $N(U, V)$ regardless of which points in $\mathbb{T}^2$ these balls are drawn around. $N^*$ will presumably depend on $\epsilon$ and some data about the matrix $A$, but should otherwise be independent of the choice of the balls $U$ and $V$. 