Math 229, Fall 2014
Project: Radioactive decay chains

The simple radioactive decay model in which one element changes into another is modeled by an exponential DE as discussed in class. However, these simple transitions do not occur in isolation in nature. Most radioactive decay occurs through radioactive decay chains involving many steps.

As a simple model, suppose we have a 2-step decay chain \( A \overset{k_1}{\rightarrow} B \overset{k_2}{\rightarrow} C \) starting with element A and ending with C. The constants \( k_1 \) and \( k_2 \) are the half-lives for these decays. This can be modeled with a system of first-order DEs for the amounts of A, B and C.

In your writeup, please include the following:

1. For the two-step decay, write and solve a system of 3 first-order DEs.

2. Suppose that you start with 100 moles of A. How long does it take until you have 50 moles of A? 50 moles of C? How long again until 25 moles of C? 12.5 moles of C?

3. Based on your answer to the previous problem, is there an ‘effective half-life’ for the chain as a whole? Are there different ways to define half-life in this case?

4. Some decay series go through multiple paths. For example:

Here each atom of V decays to Z either along the path \( V \rightarrow X \rightarrow Y \rightarrow Z \), or \( V \rightarrow W \rightarrow Z \); the first path happens with probability \( p \) and the second with probability \( 1 - p \). Write and solve a linear system for this decay chain.

5. Apply your model to the Thorium series, which you can find detailed here:
Write out the linear system, and use a computer to find solutions for some simple initial conditions. Using your previous work, what should the ‘effective half-life’ for this chain be?

6. Further up on that page, Wikipedia claims: “When equilibrium is achieved, a granddaughter isotope is present in direct proportion to its half-life”. Does this hold for your models? If not, can you find a way to make sense of this statement?

7. If you are working with Thorium in the lab, do you need to account for the other elements in this decay chain?