In this project, you will model and study coupled springs, or coupled oscillators. You will then use your model to predict resonance frequencies for some simple linear molecules, like CO$_2$.

Consider three masses ($m_1$, $m_2$, and $m_3$) arranged in order along the $x$-axis. Springs with spring constants $k_1$ and $k_2$ connect $m_1$ to $m_2$ and $m_2$ to $m_3$, respectively, as shown in the picture below.

We will assume that there are no external forces on this system, so its center of mass always remains at $x = 0$. This also means that at all times, the velocities of the masses are such that the total momentum of the three masses is zero. We will also assume the masses only move in the $x$-direction, so there is no bending of the springs.

Construct a system of DEs to model this situation. It will be useful to use variables $x_1$, $x_2$, $x_3$ which measure how far the three masses are away from their equilibrium (i.e. springs not stretched or compressed) positions. You should then be able to use the center of mass condition mentioned above to find some relations between $x_1$, $x_2$, and $x_3$.

In your writeup, please include the following:

1. Describe your model for the 3-mass system and how you arrived at it.

2. Solve the system you found above. Our assumptions about the center of mass and total momentum may be helpful.

3. Some sin and cos terms should have arisen in your solutions. What are the frequencies of these functions in terms of the masses and spring constants? There are two ‘modes’ of molecular vibration commonly identified for these solutions – symmetric and anti-symmetric. How do you see these modes in your solutions?
4. This three mass system is a model for carbon dioxide, which is a linear molecule with two Oxygen masses around one Carbon mass. Look up the molecular weights for C and O and the vibrational spectrum for CO$_2$. From this information and your model, can you predict the spring constant for the C=O double bonds? See if your prediction matches the literature.

5. Find some other linear molecule with more than three atoms. Find a model for its linear vibrations. See if you can find information about its vibrational spectrum. How many modes will it have? Can you form a prediction about the number of modes from your models?